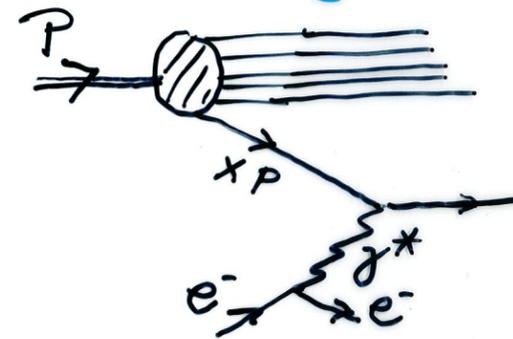


# High Energy Hard QCD

## 1. The Parton Model

### A. Traditional picture of the proton Bjorken Frame



$$P_\mu = (p + \frac{M_p^2}{2p}, 0, 0, p)$$

$$q_\mu = (q_0, q_\perp, 0)$$

$$-q_\mu q^\mu = Q^2 \approx q_\perp^2$$

$$x = \frac{Q^2}{2P \cdot q} \approx \frac{q_\perp^2}{2P q_0}$$

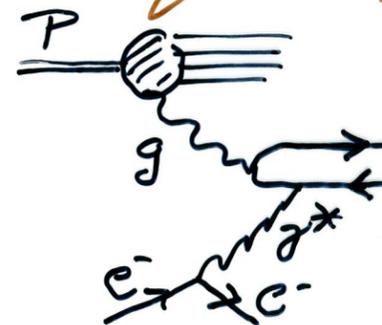
$p \rightarrow \infty$   
 $q_0 \rightarrow 0$

Photon makes "instantaneous" measurement of charged constituents of proton.

$$F_2(x, Q^2) = \sum_f e_f^2 [x q_f(x, Q^2) + x \bar{q}_f(x, Q^2)]$$

Quark local in  $x_\perp$  ( $\Delta x_\perp \sim 1/Q$ ); one layer in  $z$ .

Gluons can be measured, say, from heavy quark production.

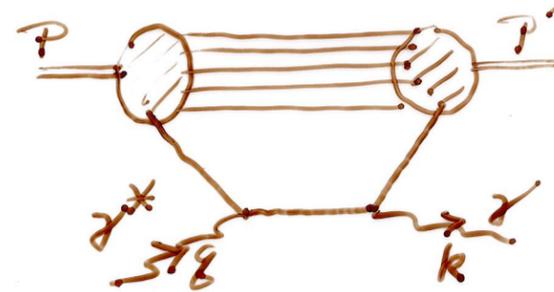


$$\sigma \sim x G(x, M^2) \cdot H_{g \gamma^* \rightarrow Q \bar{Q}}$$

## B. Refining the measurements

### B.1 Transverse spatial distributions

One can measure, in principle, the transverse coordinate distributions of the quarks and gluons.

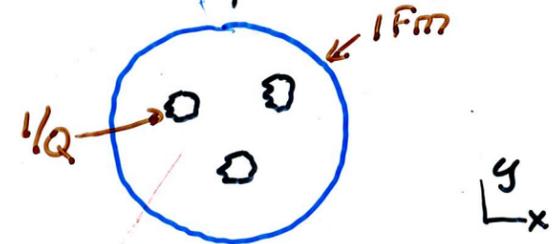


$$\gamma^*(q) \rightarrow \gamma(k)$$

$$q_{\perp} - k_{\perp} = \Delta_{\perp}$$

Fourier transform  $e^{i\Delta_{\perp} \cdot b_{\perp}}$  converts momentum amplitude to spatial amplitude,

$$g(x, b_{\perp}, Q).$$



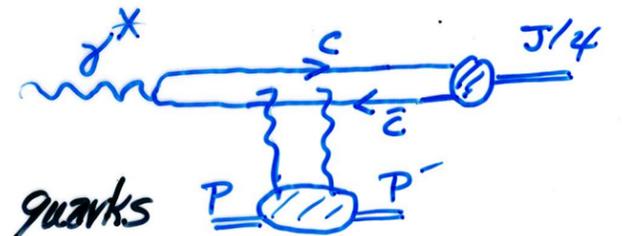
Major Focus of Jlab 12 GeV program.

Similarly,  $\gamma^*(q) \rightarrow J/\psi, \rho, \dots$  gives impact parameter dependent gluon distribution

HERA data indicate gluons more central than quarks

$$\langle r_{\perp}^2 \rangle_g \approx \frac{2}{3} \langle r_{\perp}^2 \rangle_q$$

Dedicated EIC experiments would give refined measurements for protons and nuclei.



### B.2 Spin

$g_1^p(x, Q^2)$  gives the longitudinal spin distributions in polarized proton deep inelastic scattering

% of protons spin carried by quark spin is  $\approx 25\%$

### A crisis?

No Fundamental problem. There is also

Gluon spin (Small as seen by RHIC, COMPASS)

Orbital angular momentum of quarks  $J_{lab}$   
" " " " " gluons EIC

### Why the Fuss?

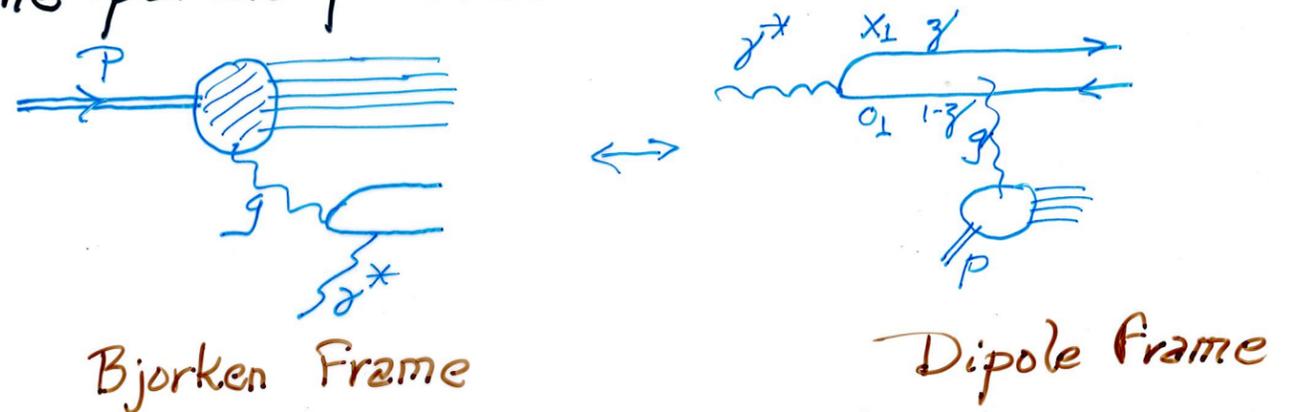
Problem with constituent quark model.

$\langle B | j_\mu | B \rangle$  and  $\langle B | j_\mu^{KS} | B \rangle$  can reasonably be evaluated in the constituent quark model.

For  $(P | j_\mu^{singlet} | P)$  constituent quark model fails!

## 2. A Different View of the Probe: The Dipole Model

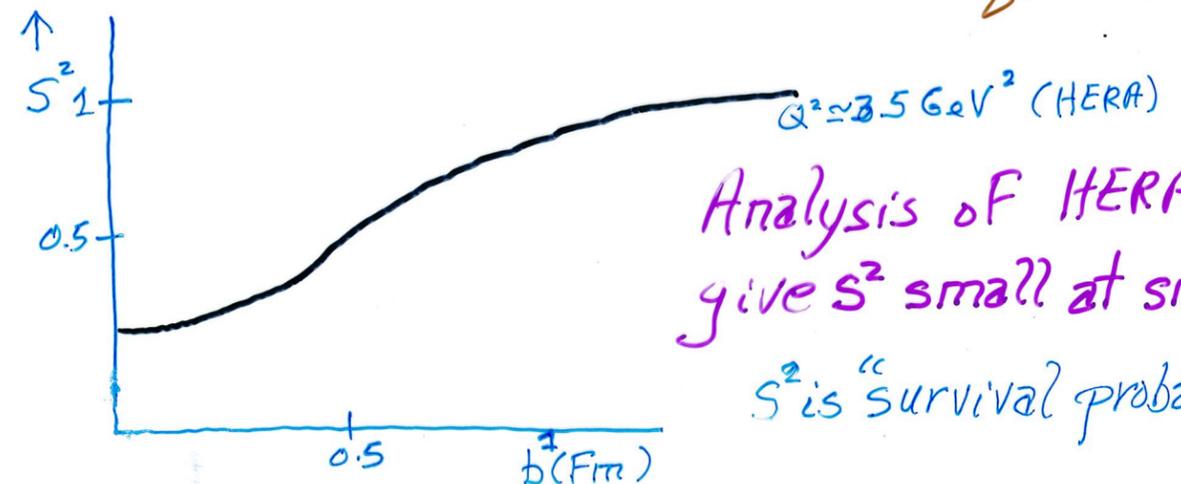
At high energy (small-x) there is a picture of DIS which is complementary (dual) to the parton picture.



$$F_2 = \sum_f e_f^2 x [q_f(x, Q^2) + \bar{q}_f(x, Q^2)] = \frac{Q^2}{4\pi^2 \alpha_{em}} \int_0^1 dx_1 \int_0^1 dz \left[ \frac{1}{z} \left| \psi_T(x_1, z, Q) \right|^2 \int d^2b_1 \underbrace{2(1-S(x_1, b_1))}_{\sigma_{dip}(x_1, b_1)} \right]$$

Unitarity limit,  $S(x_1, b_1, x) = 0$ , corresponds to

Saturation in wavefunction of proton:  $f_g \sim 1$   
 $f_q \sim 1/\alpha_s$



Analysis of HERA data give  $S^2$  small at small  $b$   
 $S^2$  is "survival probability"

### 3 Nuclear Targets

Focus on small  $x$ .

Nuclei enhance parton densities

$$XG_A(b_1, x, Q^2) = \frac{3}{2} \sqrt{1 - \frac{b_1^2}{R^2}} \frac{A}{\pi R^2} XG(x, Q^2)$$

Compare HERA for proton with EIC for nuclei

30 GeV( $e^-$ ) × 1000 GeV(P)

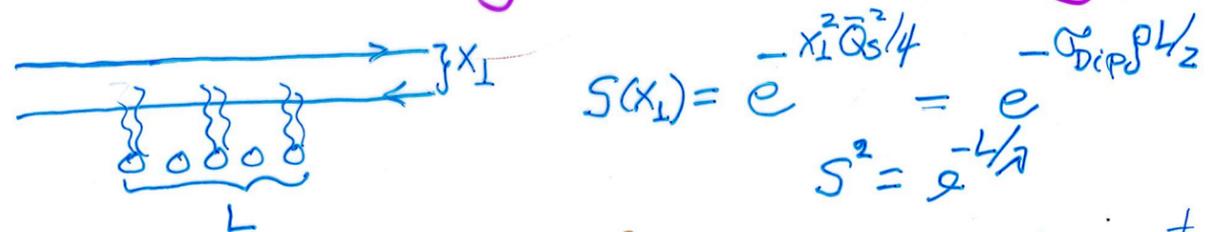
10 GeV( $e^-$ ) × 100 GeV(P)

$$\frac{X_{EIC}}{X_{HERA}} \approx 30$$

Growth in occupation  $\sim X^{-0.25}$

$$\frac{XG(x, Q^2)|_{HERA}}{XG_A(x, Q^2)|_{EIC}} \approx \frac{(30)^{0.25}}{A\text{-dependence}} \approx \frac{2.5}{A\text{-dep}} \approx 1$$

Measure of scattering in Nuclei (McLerran-Venugopalan model)



$$S(x_1) = e^{-x_1^2 \bar{Q}_s^2 / 4} = e^{-\hat{q} L P / 2}$$

$$S^2 = e^{-L/\lambda}$$

$$\bar{Q}_s^2 = \frac{C_F}{N_c} Q_s^2$$

$$Q_s^2 = \frac{4\pi\alpha_s N_c}{N_c^2 - 1} L P XG(x, 1/x_1) = \hat{q} L$$

transport coefficient

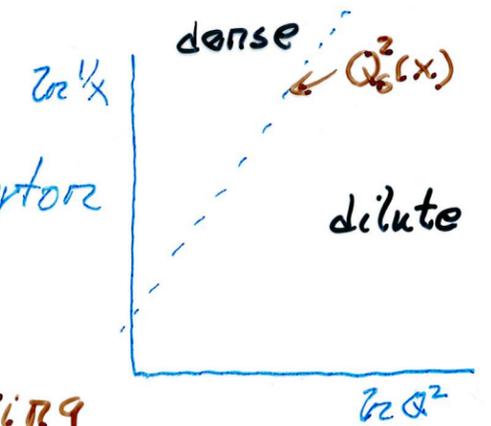
$$Q_s^2 \approx 1 \text{ GeV}^2 \text{ For RHIC, } b \approx 0$$

$$\hat{q} \approx \frac{1}{25} \text{ GeV}^2/\text{fm} \text{ For quarks at fixed target energies}$$

# 4. Going Toward High Density

## A. Theory

Scattering amplitudes, parton densities grow with energy



$F_2(x, Q^2)$ : Two ways of evolving

(a) Evolve in  $\ln Q^2$ . If  $1/x$  large must resum  $(\alpha \ln 1/x)^k$  series in all anomalous dimension functions.

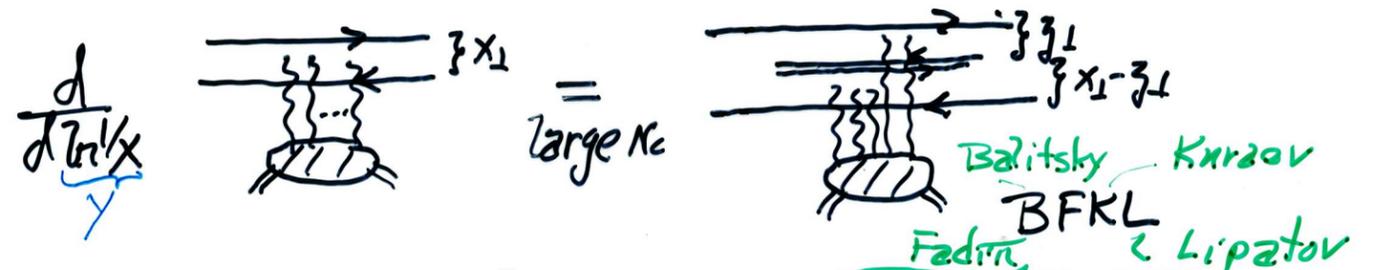
(b) Evolve in  $\ln 1/x$ . If  $\ln Q^2$  large must resum in  $\ln Q^2$ .

$Q^2$ -evolution:   $\rightarrow$  very dilute

$x$ -evolution:   $\rightarrow$  dense

Reaching high density requires strong  $x$ -evolution, that is very small  $x$ .

# Small-x evolution: Dipole-Proton and Dipole-Nucleus Scattering



$$\frac{dT(x_1, y)}{dy} = \frac{\alpha N_c}{2\pi^2} \int d^2z_1 \frac{x_1^2}{z_1^2(x_1-z_1)^2} \left[ T(z_1, y) + T(x_1-z_1, y) - T(x_1, y) - \underbrace{T^{(2)}(x_1-z_1, z_1, y)}_{\text{mean field } T(x_1-z_1, y) T(z_1, y)} \right]$$

Balitsky Kovchegov

This is BK equation

For scattering on large nucleus mean field is good and BK correct in large  $N_c$  limit. For single event evolution also ok.

BK in same universality class as Fisher Petrowsky FKPP equation.

Kolmogorov Piskounov

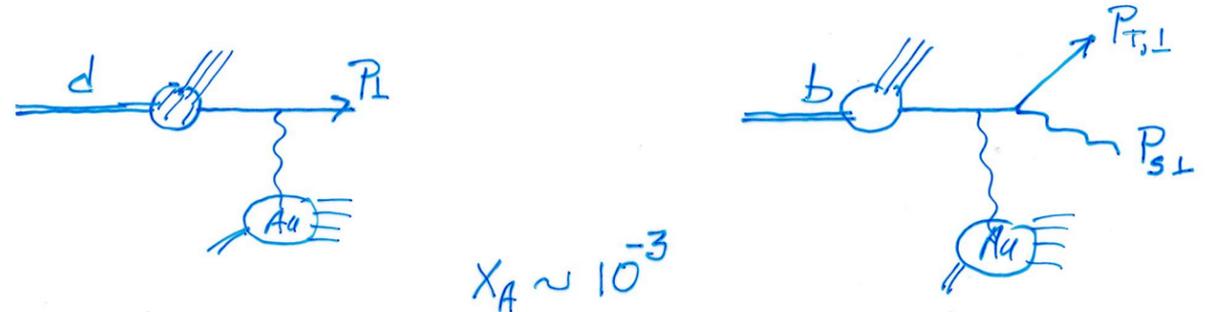
$$\frac{\partial u(x, t)}{\partial t} = \frac{\partial^2 u(x, t)}{\partial x^2} + u(x, t) - u^2(x, t)$$

$T, u = 0$  unstable Fixed point     $T, u = 1$  stable Fixed point

Approach to unitarity,  $y$  and  $x_1$  dependence of  $T(x_1, y)$  close to solved problem. Essentially the same issues as in the behavior of stochastic travelling waves.

## B. Phenomenology

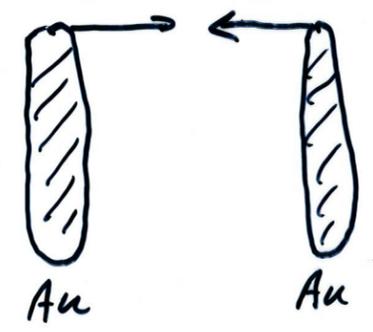
Exciting analyses of "high- $P_T$  particles in forward d-Au collisions at RHIC.



Suppression

back-to-back recoils lost in central collisions

### 5. Freeing the gluons; ion-ion collisions.



The collision can be viewed as freeing gluons in the wavefunction leading to an initial distribution of gluons  $\frac{dN^{in}}{dy} \Big|_{y=0}$  in the central region

$$\frac{dN^{in}}{dy} \sim \frac{1}{\alpha_s} Q_s^2(y) R_A^2$$

In going from RHIC to LHC the growth in particle multiplicity estimated by  $y$ -dependence of  $Q_s^2(y)$ . A bit too low suggesting some increase in parton evolution after collision.

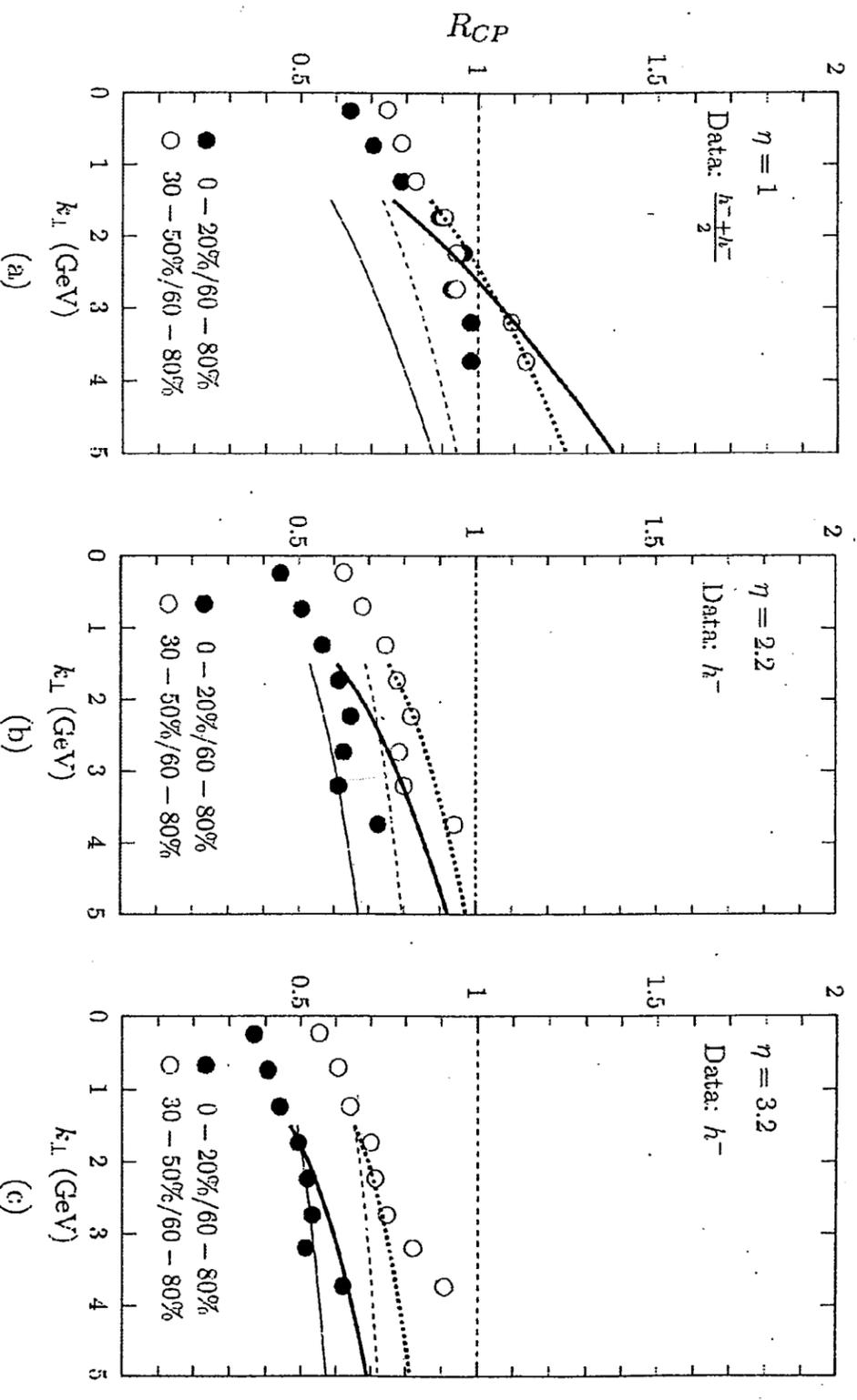
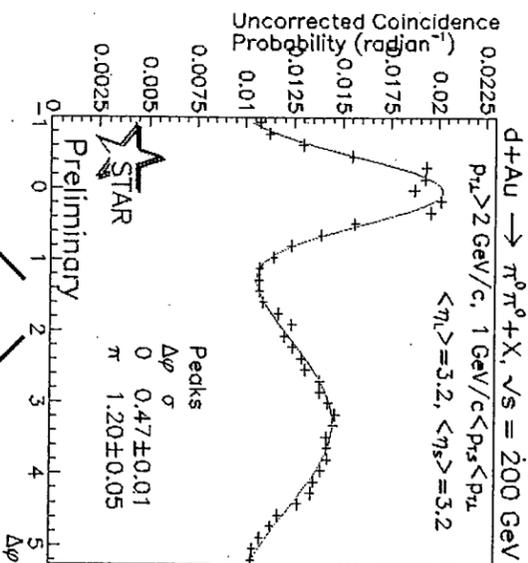
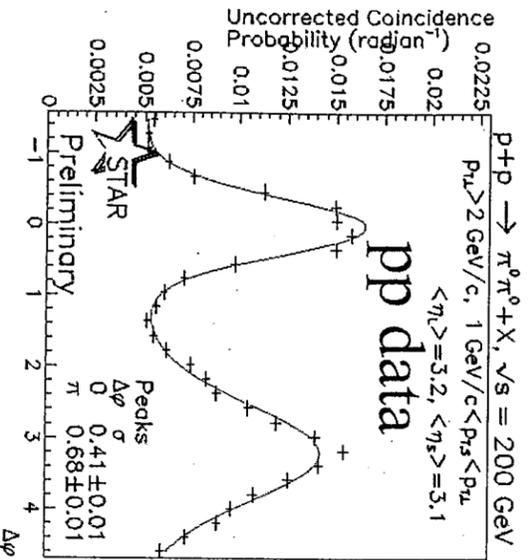


Fig. 4.  $R_{CP}$  for the BK parametrization (thick lines) and the BFKL + saturation form (thin lines) at different rapidities  $\eta = 1, 2.2$  and  $3.2$ . Full lines correspond to central over peripheral collisions (full experimental dots). Dashed lines correspond to semi-central over peripheral collisions (empty experimental dots). Data from [2].

# Forward-Forward: Centrality dependence

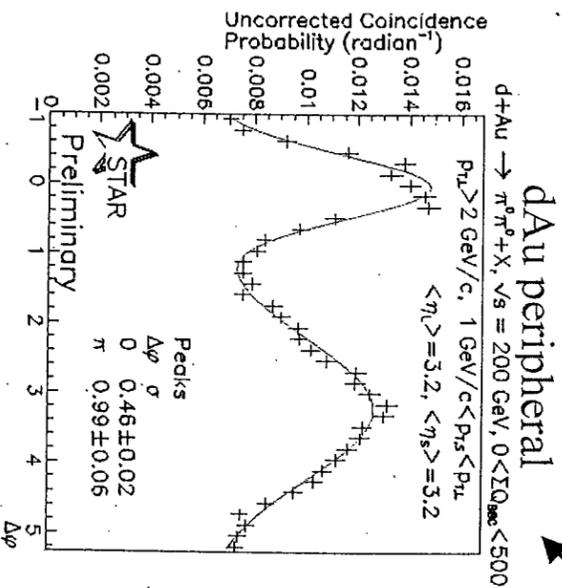


dAu all  
data

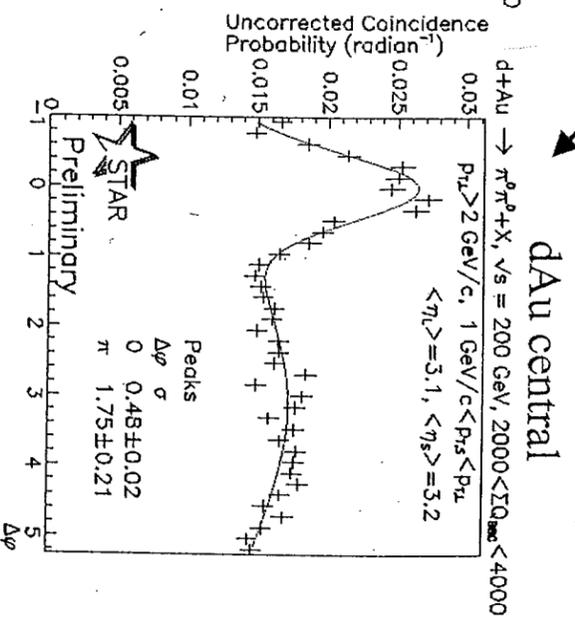
Away-side peaks  
evident in peripheral  
dAu and pp.

Near side peaks  
unchanged in dAu for  
peripheral to central.

Azimuthal  
decorrelations show  
significant dependence  
on centrality.



dAu peripheral



dAu central

Heavy ion phenomenology rich and evolving

RHIC

Strong Flow; both radial and elliptic

Large energy loss but

$$\frac{1}{2} \text{GeV/fm} \lesssim \hat{q} \lesssim 15 \text{ GeV}^2/\text{fm}$$

indicate big uncertainties

Early equilibration,  $\tau_{eq} \lesssim 1 \text{ fm}$

Hydrodynamics works well; small  $\eta/s$ ?

Is the plasma effectively strongly coupled?

LHC

Just started

Seems to confirm RHIC picture  
but quantitative comparisons  
is progress

Theoretical challenges

How to understand equilibration

Complicated scenarios

initial gluons  $\xrightarrow{\text{instability}}$  soft gluons  $\rightarrow$

turbulence  $\rightarrow$  equilibrium

### 5 Can AdS/CFT correspondence be useful

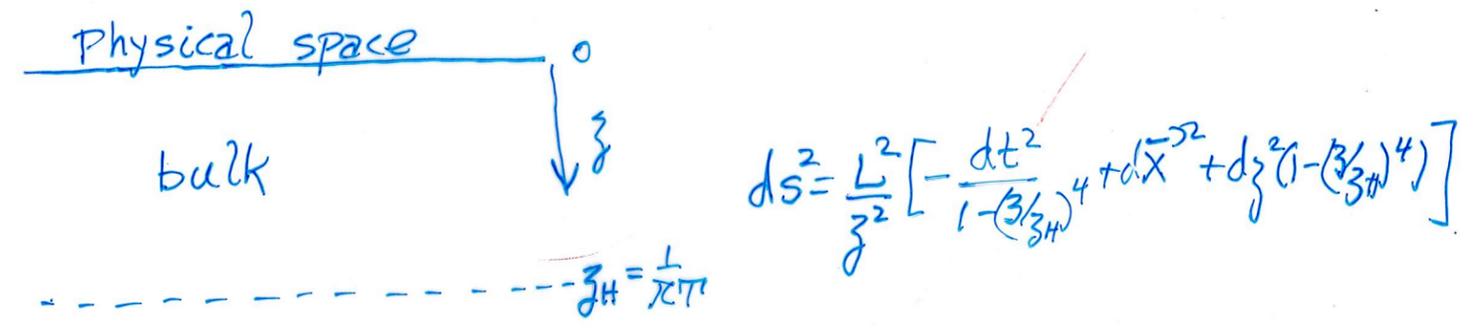
Applies to conformal Field theories

like  $N=4$  SYM.

$N=4$ SYM	gauge bosons	$\mathbb{H}$ 16.2	all in adjoint color representation
	Fermions	4.2	
	scalars	6.4	

$N=4$  SYM  $\sim$  type IIB string theory  
in  $AdS_5 \times S^5$

When  $g^2 \rightarrow 0, \lambda \rightarrow \infty$  string theory  $\rightarrow$  classical  
supergravity



Classic calculation:  $\eta/s = 1/4\pi$  Son et al

Naturally has short equilibration time,  
Strong Flow, strong jet quenching

## Unusual properties

Medium not partonic, but can be viewed as limit of strongly evolving partons

Only  $T_{\mu\nu}$  survives in hard scattering off plasma

Medium not stochastic

$\Gamma_1$ -broadening only due to radiation  
no multiple scattering  $\Gamma_2$ -broadening

Challenge in using AdS/CFT is to match hard scales (perturbative QCD) to soft scales (nonperturbative) and try to use AdS for the soft scales.